Phonon networks with SiV centers in diamond waveguides

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We propose and analyze a novel realization of a solid-state quantum network, where separated silicon-vacancy centers are coupled via the phonon modes of a quasi-1D diamond waveguide. In our approach, quantum states encoded in long-lived electronic spin states can be converted into propagating phonon wavepackets and be reabsorbed efficiently by a distant defect center. Our analysis shows that under realistic conditions, this approach enables the implementation of high-fidelity, scalable quantum communication protocols within chip-scale spin-qubit networks. Apart from quantum information processing, this setup constitutes a novel waveguide QED platform, where strong-coupling effects between solid-state defects and individual propagating phonons can be explored at the quantum level.

Electronic and nuclear spins associated with defects in solids comprise a promising platform for the realization of practical quantum technologies [1]. A prominent example is the nitrogen-vacancy (NV) center in diamond [2, 3], for which techniques for state detection [4], coherent manipulations [5–7] and local entanglement operations [8–10] have been demonstrated and employed, for example, for various nanoscale sensing applications [11]. Despite this progress in the local control of spin qubits, integrating many spins into larger networks remains a challenging task. To achieve this goal, several schemes for interfacing spins via mechanical degrees of freedom have recently been discussed [12–17] and first experiments demonstrating magnetic [18–20] or strain-induced [21–25] couplings of mechanical vibrations to both long-lived spin states and electronic excited states of NV centers have been carried out. However, the weak intrinsic coupling of spins to vibrational modes and the short coherence of optically excited states make the extension of these methods into the quantum regime challenging.

In this Letter we describe the implementation of a phonon quantum network, where negatively-charged silicon-vacancy (SiV) centers are coupled via propagating phonon modes of a 1D diamond waveguide [26–29]. The electronic ground state of the SiV center features both spin and orbital degrees of freedom [30–32], which makes it naturally suited for this task; quantum states can be encoded in long-lived superpositions of the two lowest spin-orbit-coupled states [33–37], while a controlled admixing of higher orbital states, which are susceptible to strain, gives rise to a strong and tunable coupling to phonons. The central phonon frequency of approximately 46 GHz set by the large spin-orbit splitting enables quantum-coherent operations already at convenient temperatures of $T \lesssim 1$ K, when thermal excitations at this frequency are frozen out. Our analysis shows that high-fidelity quantum state transfer protocols between distant SiV centers can be implemented under realistic conditions. Moreover, we propose a scalable operation of such phonon networks using switchable single-defect mirrors.

Model.—We consider a system as depicted in Fig. 1, where an array of SiV defects is embedded in a 1D phonon waveguide. The inset shows the level structure of the electronic ground state of the SiV center. A tunable Raman process involving the excited state $|3\rangle$ is used to coherently convert the population of the stable state $|2\rangle$ into a propagating phonon, which can be reabsorbed by any other selected center along the waveguide. See text for more details.

![FIG. 1: Setup. An array of SiV defects is embedded in a 1D phonon waveguide. The inset shows the level structure of the electronic ground state of the SiV center. A tunable Raman process involving the excited state $|3\rangle$ is used to coherently convert the population of the stable state $|2\rangle$ into a propagating phonon, which can be reabsorbed by any other selected center along the waveguide. See text for more details.](image-url)
netic field $\vec{B} = B_0 \hat{e}_z$, the Hamilton operator for a single SiV center is $(\hbar = 1)$
\begin{align}
H_{\text{SiV}} &= \omega_B |2\rangle \langle 2| + \Delta |3\rangle \langle 3| + (\Delta + \omega_B) |4\rangle \langle 4| \\
&\quad + \frac{1}{2} \left[ \Omega(t) e^{i\omega_d t + \theta(t)} |2\rangle \langle 3| + |1\rangle \langle 4| + \text{H.c.} \right],
\end{align}

where $\omega_B = \gamma_s B_0$ and $\gamma_s$ is the spin gyromagnetic ratio. In Eq. (1), we have included a time-dependent driving field with a tunable Rabi-frequency $\Omega(t)$ and phase $\theta(t)$, which couples the lower and upper states of opposite spin. This coupling can be implemented either directly with a microwave field of frequency $\omega_d \sim \Delta$ [38], or indirectly via an equivalent optical Raman process [39]. The latter method is already used in experiments to initialize and prepare individual SiV centers in superpositions of $|1\rangle$ and $|2\rangle$ [33–35] with coherence times that can exceed 10 ms in the absence of thermal processes and with dynamical decoupling [36]. Further details on the derivation of $H_{\text{SiV}}$ are given in the supplementary material [39].

For the waveguide, we consider a quasi-1D geometry of width $w$ and length $L \gg w$. The waveguide supports travelling phonon modes of frequency $\omega_{n,k}$ and mode function $\tilde{u}_{n,k}(\vec{r}) \sim \tilde{u}_{n,k}(y,z) e^{ikx}$, where $k$ is the wavevector along the waveguide direction, $n$ is the branch index and $\tilde{u}_{n,k}(y,z)$ is the transverse profile of the displacement field. The phonons induce transitions between the orbital states $|e_{\pm}\rangle$ [40–42], and the Hamiltonian for the whole system reads
\begin{align}
H &= \sum_j H_{\text{SiV}}^{(j)} + \sum_{n,k} \omega_{n,k} a_{n,k}^\dagger a_{n,k} \\
&\quad + \frac{1}{\sqrt{L}} \sum_{j,k,n} \left( g_{n,k}^j J_\lambda^j a_{n,k} e^{ikx} + \text{H.c.} \right),
\end{align}

where $j$ labels the SiV centers located at positions $\vec{r}_j = (x_j, y_j, z_j)$, $J_- = (J_+)^\dagger = |1\rangle \langle 3| + |2\rangle \langle 4|$ is the spin-conserving lowering operator and $a_{n,k}$ ($a_{n,k}^\dagger$) are the annihilation (creation) operators for the phonon modes. The couplings $g_{n,k}^j \equiv g_{n,k}(y_j, z_j)$ depend on the components of the local strain tensor, $\epsilon_{n,k}^{\alpha\beta}(\vec{r}_j) = \frac{1}{2} \frac{\partial}{\partial x} \tilde{u}_{n,k}^{\alpha}(\vec{r}_j) + \frac{\partial}{\partial z} \tilde{u}_{n,k}^{\beta}(\vec{r}_j)$, and can be evaluated for a known transverse mode profile $\tilde{u}_{n,k}^\alpha(y,z)$ [39, 42]. We express the resulting couplings as
\begin{align}
g_{n,k}^j &= d \sqrt{\frac{\hbar k^2}{2\rho A \omega_{n,k}}} \xi_{n,k}(y_j, z_j),
\end{align}

where $d/2\pi \sim 1$ PHz is the strain sensitivity of the orbital states [40, 41], $\rho$ the density and $A$ the transverse area of the waveguide. The dimensionless coupling profile $\xi_{n,k}(y,z)$ accounts for the specific strain distribution and $\xi(y,z) = 1$ for a homogeneous compression mode.

**From cavity to waveguide QED.—** For small structures, $L \sim 10 - 100 \mu m$, $w \lesssim 200$ nm, and group velocities $v \sim 10^4$ m/s, the individual phonon modes are well separated in frequency, $\Delta \omega/2\pi \gtrsim 50$ MHz, and the SiV centers can be coupled to a single standing-wave mode with a strength $g_L = g_0 \sqrt{\lambda/L} \approx 2\pi \times (4 - 14)$ MHz, where $g_0/2\pi \approx 105$ MHz and $\lambda \approx 200$ nm is the phonon wavelength. The system dynamics is then governed by a Jaynes-Cummings-type interaction between phonons and orbital states [39]. In the strong coupling regime, $g_L > \kappa = \Delta/Q$, which is reached for moderate mechanical quality factors of $Q > 10^4$, a coherent exchange of phonons and defect excitations becomes possible. For longer waveguides, the coupling to the quasi-continuum of phonon modes is characterized by the resulting decay rate $\Gamma_j(\Delta) = \sum_n \Gamma_{j,n}(\Delta)$ for states $|3\rangle$ and $|4\rangle$, where
\begin{align}
\Gamma_{j,n}(\omega) = \lim_{L \to \infty} \frac{2\pi}{k} \sum_k |g_{n,k}^j|^2 \delta(\omega - \omega_{n,k}).
\end{align}

For a single compression mode with $\tilde{u}^+ \sim \vec{x}$ and a linear dispersion $\omega_k = vk$, we obtain $\Gamma(\omega) = d^2 v \hbar a / (\rho Av^3)$, which results in a characteristic phonon emission rate of $\Gamma(\Delta)/2\pi \sim 1$ MHz [42].

**FIG. 2: Phonon waveguide.** (a) Acoustic dispersion relation for a triangular waveguide of width $w = 130$ nm and etchangle $\varphi = 35^\circ$. Symmetric (solid lines) and anti-symmetric (dashed lines) branches with respect to the vertical mirror-symmetry plane are shown. (b) Normalized displacement profiles of the symmetric phonons at 46 GHz. (c) The emission rates into the symmetric longitudinal ($\Gamma_L$) and transverse ($\Gamma_T$) polarization modes at 46 GHz are plotted for different positions of the SiV center within the triangular cross-section. (d) $\Gamma_T$ and fraction $(\beta_T)$ of spontaneous emission into the longitudinal branch for different positions of the SiV center along the vertical mirror-symmetry axis. For all results, an orientation of the waveguide along the [110] crystal axis of diamond and SiV centers oriented along [111] and [111], i.e., orthogonal to the waveguide axis, have been assumed.
waveguide [26, 40] of width \( w = 130 \) nm. The SIV centers couple primarily to a longitudinal (\( l \)) compression and a transverse (\( t \)) flexural mode with group velocities \( v_l = 1.71 \times 10^4 \) m/s and \( v_t = 0.73 \times 10^4 \) m/s, respectively. The coupling to the other two branches of odd symmetry can be neglected for defects near the center of the waveguide. Fig. 2(c) and (d) show that the rates \( \Gamma_{tt} \) are quite insensitive to the exact location of the SIV center. However, the fraction of phonons emitted into a specific branch, \( \beta_n = \Gamma_n/\Gamma \), is significantly below unity as emission is split between a pair of modes. In optical waveguides [43], a value of \( \beta < 1 \) usually arises from the emission of photons into non-guided modes, which are irreversibly lost. For a phonon waveguide this is not the case, but the multi-branch nature of the waveguide must be fully taken into account. In all examples below we assume \( \beta_l = \beta_t = 0.5 \), which is most relevant for SIV defects located near the center of the beam.

**Coherent spin–phonon interface.**—We are interested in the transfer of a qubit state, encoded into the stable states \( |1 \rangle \) and \( |2 \rangle \), between an arbitrary pair of emitting (\( e \)) and receiving (\( r \)) defects in the waveguide,

\[
|\alpha(1)\rangle_e + \beta|2\rangle_e|0\rangle_r \rightarrow |1\rangle_e|\alpha(1)\rangle_r + \beta|2\rangle_r. \tag{5}
\]

As shown in Fig. 1, this can be achieved by inducing a Raman transition via state \( |3 \rangle \) to convert the population in state \( |2 \rangle \) into a propagating phonon and by reverting the process at the receiving center. For low enough temperatures, \( T \ll \hbar \Delta/k_B \approx 2.2 \) K, such that all phonon modes are initially in the vacuum state, this scenario is described by the following ansatz for the wavefunction \( |\psi(t)\rangle = |\alpha(1)\rangle|1\rangle + |\beta(2)\rangle|0\rangle \), where \( |1\rangle, |0\rangle \) is the ground state with all SIV centers in state \( |1 \rangle \) and \( C^d(t) = \sum_{j=1}^{n,t} e^{i\omega_n t} |j\rangle(t) \langle j| + b_j(t) e^{-i\omega_n t} |j\rangle(t) \langle j| \) creates a single excitation distributed between the SIV centers and the phonons. The central phonon frequency \( \omega_0 = \Delta_l + \delta_j \) is assumed to be fixed by compensating small inhomogeneities in the \( \Delta_l \) by the detunings \( \delta_j = \omega_0^j - (\Delta_j - \omega_0^j) \).

By adiabatically eliminating the fast decaying amplitudes \( b_j(t) \), we derive effective equations of motion for the slowly varying amplitudes \( c_j(t) \). From this derivation, detailed in [39], we obtain for each qubit amplitude

\[
\dot{c}_j(t) = -\gamma_j(t)/2 c_j(t) - \sum_n \sqrt{\gamma_{j,n}(t)}/2 e^{-i\theta_{j,n}(t)} \Phi_{j,n}^\text{in}(t), \tag{6}
\]

where \( \gamma_j(t) = \sum_n \gamma_{j,n}(t) \) is the effective decay rate of state \( |2\rangle_n \) and

\[
\gamma_{j,n}(t) = \frac{\Omega_j^2(t)}{4\delta^2 + \Gamma_j^2(\omega_0)} \Gamma_{j,n}(\omega_0). \tag{7}
\]

Assuming \( 0 \leq \Omega(t)/2\pi \leq 70 \) MHz and \( \delta/2\pi = 100 \) MHz, this rate can be tuned below \( \gamma_j = 0 \) and a maximal value of \( \gamma_{\text{max}}/2\pi \approx 250 \) kHz, which is still fast compared to the expected bare dephasing times \( T_2^* \approx 10 - 100 \mu s \) of the qubit state [36]. At the same time, the large detuning \( \delta \gg \Gamma(\Delta) \) ensures that any residual scattering of phonons from an undriven defect is strongly suppressed [39].

The last term in Eq. (6), where \( \Phi_{j,n}^\text{in} = \Phi_{j,n}^\text{in,L} + \Phi_{j,n}^\text{in,R} \), describes the coupling of an SIV center to the left- (L) and right- (R) incoming fields \( \Phi_{j,n}^\text{in,L/R} \), which themselves are related to the corresponding outgoing fields by [44]

\[
\Phi_{j,n}^\text{out,R/L}(t) = \Phi_{j,n}^\text{in,R/L}(t) + \sqrt{\gamma_{j,n}(t)}/2 c_j(t) e^{i\theta_{j,n}(t)}. \tag{8}
\]

Together with Eq. (6), these input-output relations specify the local dynamics at each node and must be supplemented by a set of propagation relations for all fields [cf. Fig. 3(a)]. As an example, for \( x_r > x_e \), the right propagating fields obey \( \Phi_{e,n}^\text{out,R}(t) = \sqrt{R_n} \Phi_{e,n}^\text{out,L}(t) e^{i\phi_n^\text{R}} \), where \( \tau_{n,r}^\text{R} = (x_r - x_e)/v_n \) and \( \phi_n^\text{R} = k_n(x_r - x_e) \) are the respective propagation times and phases. Reflections at the boundaries lead to a retarded interaction of each center with its own emitted field. For example, \( \Phi_{e,n}^\text{in,R}(t) = -\sqrt{R_n} \Phi_{e,n}^\text{out,L}(t - \tau_{n,r}^\text{L}) e^{i\phi_n^\text{L}} \), where \( \tau_{n,r}^\text{L} = (x_e - x_r)/v_n \) and \( \phi_n^\text{L} = 2k_n x_e \), and the reflectivity \( R_n \leq 1 \) has been introduced to model losses. The combined set of time-nonlocal equations for the SIV amplitudes can be solved numerically for given positions \( x_j \) and pulses \( \gamma_{j,n}(t) \).

Since any deterministic phase acquired during the protocol can be undone by a local qubit rotation, we identify \( F(t) = |c_r(t)|^2 \) with the fidelity of the transfer, which exceeds the classical bound for \( F > 2/3 \) [45].

**Quantum state transfer.**—In Fig. 3(b) we first consider constant rates \( \gamma_{j,n}(t) = \gamma_{\text{max}}/2 \), in which case a state transfer is achieved over multiple round-trips of the emitted wave-packet. For \( L \sim 100 \) nm, the round-trip times \( 2L/v_n \) are still short compared to \( \gamma_{\text{max}} \) and we recover the standing-wave picture with splittings \( \Delta \omega_n = \pi v_n/2L \) between consecutive \( k \)-modes. When only the transverse mode is resonant, \( \Delta \omega_n = \pi v_n/2L \) between consecutive \( k \)-modes. When only the transverse mode is resonant, \( \Delta \omega_n = \pi v_n/2L \) between consecutive \( k \)-modes. When only the transverse mode is resonant, \( \Delta \omega_n = \pi v_n/2L \) between consecutive \( k \)-modes. When only the transverse mode is resonant, \( \Delta \omega_n = \pi v_n/2L \) between consecutive \( k \)-modes.

For maximal coupling \( |\phi_n^\text{L}| = 2(2m + 1)\pi \) and for maximal coupling \( |\phi_n^\text{R}| = 2m \pi \), we observe damped oscillations with a fast frequency \( \tilde{\gamma} = \sqrt{\gamma_{\text{max}} \Delta \omega_n}/2 \approx 2 \times 1.2 \) MHz and decay rate \( \kappa = -\Delta \omega_n/2 \log R \approx 2 \times 0.93 \) MHz. This result is expected from a single-mode description of the waveguide [39], and is recovered here as a limiting case of our general framework. The losses from multiple imperfect reflections at the boundaries can be partially suppressed at the expense of a slightly slower transfer by detuning the SIV centers from the closest mode by \( \delta_0 > \tilde{\gamma} \). In this case the SIV centers communicate via an exchange of virtual phonons and \( \kappa \rightarrow \kappa/(\tilde{\gamma}/\delta_0)^2 \). For a maximal detuning \( \delta_0 = \Delta \omega_n/2 \), the transfer fidelity scales approximately as \( F \approx R - \pi^2/(8\Omega_{\text{max}}^2) \) [39]. For \( T_2^* \approx 100 \mu s \) and \( R > 0.99 \), which can be achieved, for example, by phononic Bragg mirrors [46], gate fidelities of \( F \gtrsim 0.99 \) are possible.
As illustrated by the solid line in Fig. 3(b), the simple cavity picture fails for longer waveguides, where multimode and propagation effects become non-negligible. In Fig. 3(c) we illustrate a more general and more robust protocol, where the phonons ideally travel the waveguide only once. Here, the emission is gradually turned on with a fixed pulse \( \gamma_\text{in}(t)/\gamma_{\text{max}} = \min(1, e^{(t-t_\text{in})/\tau_p}) \), while \( \gamma_r(t) \) and \( \theta_r(t) \) are constructed numerically by minimizing at every time step the back-reflected transverse field \( |\Phi_{\text{out},L}| \). For slow pulses, \( \gamma_{\text{max}}/\tau_p \gg 1 \), a perfect destructive interference between the field reflected from the boundary and the field emitted by the receiving center can be achieved, i.e., \( \Phi_{\text{out},L}^{\text{in}}(t) + e^{i(\theta_r(t)/2)/\tau_p} \phi^\text{in}(t) e^{i\theta_r(t)} = 0 \). For a single branch \( (\theta_\text{in} = 1) \) this results in a complete suppression of the signal traveling back to the emitting center so that for \( R = 1 \) and negligible retardation effects, a perfect state transfer can be implemented [13, 47-49]. Fig. 3(c) shows that this approach also leads to high transfer fidelities under more general conditions, where all propagation effects are taken into account and multiple independent channels participate in the transfer. Importantly, since there are no resonances building up, this strategy is independent of \( L \) and can be applied for short and long waveguides equally well.

In the examples shown in Fig. 3(b)-(d), the SiV centers are placed at positions near the ends of the waveguide, where the effective emission rate \( \gamma_{\text{eff},L}(t) = 2\gamma_{\text{eff},L}(t)\sin^2(\phi^\text{in}/2) \) [50] into both modes is maximal. Fig. 3(e) shows the achievable transfer fidelities when the position of the receiving center is varied over several wavelengths. We observe plateaus of high fidelity extending over \( \sim 100 \) nm, interrupted by a few sharp dips arising from a complete destructive interference, i.e., \( \phi_r \approx \pi \). This position insensitivity, even in a multimode scenario, can be understood from a more detailed inspection of the outgoing fields \( \Phi_{\text{out},L}^{\text{in}}(t) \) [39] and makes the transfer protocol consistent with uncertainties of \( \delta x < 50 \) nm achieved with state-of-the-art implantation techniques [51].

**Scalability.**—In Fig. 4(a), we consider a waveguide of length \( L = 500 \mu m \) containing 49 SiV centers spaced by \( \Delta x = 10 \mu m \) to allow individual addressing by optical or microwave fields. The resulting quantum connectivity matrix, i.e., the achievable state transfer fidelity between each pair, shows that apart from a few exceptions, most centers can be connected efficiently and that in principle, the operation of large scale networks is possible. By using phononic bandstructure engineering [46, 52], single mode [53] or chiral phononic waveguides [54], the transfer fidelities can be further increased beyond the basic scenario considered here. In practice, propagation losses and elastic phonon scattering will set additional
limitations for the overall size of the network. In Fig. 4 (b), we show a general strategy to overcome these limitations by separating the whole waveguide into smaller segments using additional ‘mirror centers’. Here the two outermost SiV centers simply reflect the incoming phonon wavepacket [55], and thus create an effective cavity within the waveguide [56, 57]. This is illustrated in Fig. 4 (b), where we plot the resulting state transfer fidelity for two SiV centers localized inside this effective cavity. For transfer pulses that are long compared to $\gamma_{\max}^{-1}$, the outmost centers act as almost perfect mirrors, such that even in an infinite waveguide state transfer protocols within reconfigurable sections of the network can be implemented.

**Conclusion.**—We have shown how an efficient coupling between individual SiV centers and propagating phonons in a diamond waveguide can be realized and used for quantum networking applications. By employing direct spin-phonon couplings in the presence of a transverse magnetic field [58] or defect-phonon interactions in other materials [59–61], many of the described techniques could also be adapted for lower phonon frequencies $\sim 5 – 10$ GHz, where many advanced phononic engineering methods are already available. When combined with local control operations involving adjacent nuclear spins as quantum memories [10, 33, 62, 63], the set of all these techniques provides a realistic approach for a scalable quantum memories platform with spins in solids.

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